Characterization of interpolation between Grand, small or classical Lebesgue spaces

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Abstract

In this paper, we show that the interpolation spaces between Grand, small or classical Lebesgue are so called Lorentz-Zygmund spaces or more generally $G\Gamma$ -spaces. As a direct consequence of our results any Lorentz-Zygmund space $L^{a,r}(LogL)^{\beta}$, is an interpolation space in the sense of Peetre between either two Grand Lebesgue spaces or between two small spaces provided that $1 < a < \infty, \beta \neq 0.$ The method consists in computing the so called K-functional of the interpolation space and in identifying the associated norm.

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