On the basis of the space of generalized theta-series for quadratic forms of five variables

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In this paper the spherical polynomials of order ν with respect to quadratic form of five variables are constructed. The vector space of theta-series $T(\nu, Q)$ is considered and the basis of these space is constructed.

For quadratic form

$Q_1(x) = Q_1(x_1, x_2, \dots, x_5) = b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 + b_{12}x_1x_2$

we have proved the following

Theorem The generalized theta-series:

$$\vartheta(\tau, P_1, Q_1) = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n} P_1(x)\right)z^n = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n}\left(\frac{b_{12}}{4b_{22}}x_1^2 + x_1x_2\right)\right)z^n =$$

$=\frac{b_{12}}{2b_{22}}z^{b_{11}}+\ldots+0\,z^{b_{22}}+\ldots+0\,z^{b_{33}}+\ldots+0\,z^{b_{44}}+\ldots+0\,z^{b_{55}}+\ldots,$

$$\vartheta(\tau, P_5, Q_1) = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n} P_5(x)\right)z^n = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n}\left(-\frac{b_{11}}{b_{22}}x_1^2 + x_2^2\right)\right)z^n =$$

$=-\frac{2b_{11}}{2b_{22}}z^{b_{11}}+\ldots+2\,z^{b_{22}}+\ldots+0\,z^{b_{33}}+\ldots+0\,z^{b_{44}}+\ldots+0\,z^{b_{55}}+\cdots,$

$$\vartheta(\tau, P_9, Q_1) = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n} P_9(x)\right)z^n = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n}\left(-\frac{4b_{11}b_{22}-b_{12}^2}{4b_{22}b_{33}}x_1^2 + x_3^2\right)\right)z^n =$$

$=-\frac{4b_{11}b_{22}-b_{12}^2}{2b_{22}b_{33}}z^{b_{11}}+\ldots+0\,z^{b_{22}}+\ldots+2\,z^{b_{33}}+\ldots+0\,z^{b_{44}}+\ldots+0\,z^{b_{55}}+\cdots,$

$$\vartheta(\tau, P_{12}, Q_1) = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n} P_1(x)\right)z^n = \sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n}\left(-\frac{4b_{11}b_{22}-b_{12}^2}{4b_{22}b_{44}}x_1^2 + x_4^2\right)\right)z^n =$$

$$=-\frac{4b_{11}b_{22}-b_{12}^2}{2b_{22}b_{44}}z^{b_{11}}+\ldots+0\,z^{b_{22}}+\ldots+0\,z^{b_{33}}+\ldots+2\,z^{b_{44}}+\ldots+0\,z^{b_{55}}+\cdots,$$

$$\vartheta(\tau,P_{14},Q_1)=\sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n}P_1(x)\right)z^n=\sum_{n=1}^{\infty}\left(\sum_{Q_1(x)=n}\left(\frac{b_{12}}{4b_{22}}x_1^2+x_1x_2\right)\right)z^n$$

$$=-\frac{4b_{11}b_{22}-b_{12}^2}{2b_{22}b_{55}}z^{b_{11}}+\ldots+0\,z^{b_{22}}+\ldots+0\,z^{b_{33}}+\ldots+0\,z^{b_{44}}+\ldots+2\,z^{b_{55}}+\ldots$$

are linearly independent and they form the basis of the space $\mathrm{T}(\nu,Q_1)$.

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