ON THE wtt-REDUCIBILITY

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Friedberg and Rogers [3] defined the notion of wtt-reducibility and Tennenbaum (see, [3, p.159]) defined the notion of Q-reducibility on sets of natural numbers. Lachlan [1] defined the notion of bwtt-reducibility and in [2] it is defined the notions of bsQ-reducibility and sQ-reducibility.

Our notation and terminology are standard and can be found in [2] and [3].

We have obtained the following results.

Theorem 1. A computably enumerable (c.e.) set A is wtt-complete if and only if there exists a computable function f such that for all $x \in \omega$, $D_{f(x)} \cap (\bar{A} \,\Delta\, W_x) \neq \emptyset$, where $\bar{A} \,\Delta\, W_x = (\bar{A} - W_x) \cup (W_x - \bar{A})$.

Theorem 2. Let A be a c.e. set. Then the following statements are equivalent.

1. A is a bwtt-complete.
2. There exists a computable function f and a number n such that for all $x \in \omega$, $|D_{f(x)}| \leq n$ & $W_x \cap A = \emptyset \Rightarrow D_{f(x)} \cap \overline{W_x \cup A} \neq \emptyset$.
3. There exists a computable function g and a number m such that for all $x \in \omega$, $|D_{g(x)}| \leq m$ & $D_{g(x)} \cap (\bar{A} \,\Delta\, W_x) \neq \emptyset$.

Theorem 3. Every noncomputable c.e. bwtt-degree contains infinite many pairwise bsQ-incomparable c.e. sets.

Theorem 4. Let A be a noncomputable c.e. semicomputable set. Then for any B, $\emptyset <_{wtt} B \leq_{wtt} A$, there is a set C, such that $C \leq_{sQ} A$ & $C \equiv_{btt} B$.

Theorem 5. Every noncomputable c.e. wtt-degree contains a c.e. set A, such that the Q-degree of A contains neither simple nor nowhere simple sets.

References

[1] A.H.Lachlan, wtt-complete sets are not necessarily tt-complete, Proc. Amer. Math. Soc., 48, N 2, (1975) 429-434.

[2] R.Sh.Omanadze, Quasi-degrees of recursively enumerable sets. In: Cooper, S.B., Goncharov, S. Computability and Models: Perspectives East and West, Kluwer/Plenum, New York, Boston, Dordrecht, London, Moscow, 2004.

[3] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.