Absolutely convergence factors of Fourier series

Giorgi Tuberidze

e-mail: giorgi.tutberidze257@ens.tsu.edu.ge

[a] Department of Mathematic, Faculty Of Exact And
Natural Sciences , Ivane Javakhishvili Tbilisi State
University, Chavachavadze #1,

In 1962, A. Olevsky (see [1]) proved that if $f(x) \in L_2(0,1)$ and $(a_n) \in l_2$, then there exist orthonormal systems $(\varphi_n(x))$, such that

$$c_n(f) = c \cdot a_n, \qquad n = 1,2,\ldots,$$

and c is on absulute constant which does not depend on n. Let $f_0(x) = 1$, $x \in [0,1]$ and

$$a_n = \begin{cases} 0, & if \quad n \neq 2^k \\ \dfrac{1}{\sqrt{k}\log(k+1)}, & if \;\; n = 2^k, n = 1,2,\ldots \end{cases}$$

Then

$$\sum_{n=0}^{\infty} a_n^2 = \sum_{k=1}^{\infty} \frac{1}{k \cdot \log^2(k+1)} < +\infty$$

Then there exists an orthonormal system $(\varphi_n(x))$, such that $a_n = c \cdot c_n(f_0)$. It follows that

$$\sum_{n=1}^{\infty} \left|c_n(f_0)\right|^p = \frac{1}{|c|^p} \cdot \sum_{k=1}^{\infty} \frac{1}{(k+1)^{\frac{p}{2}} \cdot \log^p(k+1)} = +\infty.$$

for 1<p<2.

So, there exists an orthonormal system $(\varphi_n(x))$, for which

$$\sum_{n=1}^{\infty} \left|c_n(f)\right|^p < +\infty, \qquad \text{when } \; p \in (1,2).$$

Our main aim is to get such systems from generalized orthonormal systems set for which the series compiled by furrier coefficients are absolutely convergent in power of p>1 for a proper factor.