Some remarks on classes of functions of generalized bounded variation

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Definition 1. *Let* (α_n) *and* (S_n) *be a sequence of real numbers, where* $\alpha_n > 1,\ n \in \mathbb{N},$ *and*

$$\sigma_n^{\alpha_n} \equiv \sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n - 1} S_\nu / A_n^{\alpha_n}, \qquad A_k^{\alpha_n} = (\alpha_n + 1)\cdot \ldots \cdot (\alpha_n + k)/k!. \tag{1}$$

It is clear that $\sigma_n^0 = S_n$. If (α_n) is a constant sequence $(\alpha_n = \alpha,\ n \in \mathbb{N})$ then $\sigma_n^{\alpha_n}$ coincides with the usual Cesáro σ_n^α -means [3]. These means were studied by Kaplan [2]. If in (1) instead of S_ν we substitute $S_\nu(f,x)$ partial sums of the Fourier series of a function f with respect to the trigonometric system then the corresponding means σ_n^α is denoted by $\sigma_n^{\alpha_n}(f,x)$ and we shall call them by Cesáro means.

Definition 2. *Let* φ *be* an increasing sequence, $\varphi(1) \geq 2$ and $\lim\limits_{n\to\infty} \varphi(n) = +\infty$. Suppose f be a measurable 2π -periodic function defined on $(-\infty,+\infty)$. Let $p(n)$ be an icreasing sequence for wich $1 \leq p(n) \uparrow p$, $n = 1,2...,\ 1 \leq p \leq +\infty$. We say that a function f belongs to the class $B\Lambda\left(p(n)\uparrow p,\varphi\right)$ if

$$\sup_m \sup_{h \geq 1/\varphi(m)} \left\{ \frac{1}{h} \int_0^{2\pi} \left| f(x+h) - f(x) \right|^{p(m)} dx \right\}^{1/p(m)} < \infty .$$

This class is introduced in [1] and studied the properties of this class when $p = +\infty$. It is found that in the case $1 < p < +\infty$ class $B\Lambda\left(p(n)\uparrow p,\varphi\right)$ has the different properties. In particular, it is proved:

1. If $p \in (1,+\infty)$ then class $B\Lambda\left(p(n)\uparrow p,\varphi\right)$ does not depend on function φ;
2. Every class $B\Lambda\left(p(n)\uparrow p,\varphi\right)$ containes a function which is not essentially bounded.

Besides, the behaviour of generalized Cesáro means in the space of continuous functions are investigated.

References

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[3] A. Zygmund, Trigonometric series, Cambridge University Press, Vol.1 (1959).