On some properties of function's coefficients with respect to block-orthonormal systems

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Below a question connected with some properties of function's coefficients with respect to block-orthonormal systems are considered.

Let $\{N_k\}$ be increasing sequences of natural numbers and

$$\Delta_k = \left(N_k,\, N_{k+1}\right], \quad (k \geq 1) .$$

Let $\{\varphi_n\}$ be a system of functions from $L^2(0,1)$. The system $\{\varphi_n\}$ will be called a Δ_k -orthonormal system if $\|\varphi_n\|_2 = 1$, $n = 1, 2, \ldots$ and $(\varphi_i, \varphi_j) = 0$, for $(i, j) \in \Delta_k$, $i \neq j$, $(k \geq 1)$.

We introduce $k(n) = \max\{k : N_k < n\}$. Take $\{\varphi_n\}$ Δ_k -orthonormal system from $L^2(0,1)$ and for every function $f \in L^2(0,1)$ define coefficients with respect to the system $\{\varphi_n\}$:

$$c_n = c_n(f, \varphi_n, \Delta_k) = \int_0^1 f(x)\varphi_n(x)dx;$$

We studied the connection between coefficients c_n and the L^2 -norm of the function *f*. Furthermore, the conditions are established for te sequence $\{N_k\}$ when the series with respect to block-orthonormal system $\{\varphi_n\}$ and coefficients c_n

$$\sum_{n=1}^{\infty} \frac{c_n \varphi_n(x)}{\left(k(n)\right)^{\frac{1}{2}+\varepsilon} \ln(n+1)}$$

converges a. e., where $\varepsilon > 0$.