On representation of infinite Groups as Automorphism Groups of Graphs

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The subject of discussion is the representation of infinite groups as automorphism groups of graphs. This question originates from D.Konig's problem (see [1]). Using some results of the papers [2] and [3] we prove the following theorems.

Theorem 1. *Let* λ *be any infinite cardinal and* G is *a group such as* $|G|\leq\lambda$, t*hen there exists a family of pairwise non isomorphic connected graphs* $\{H_i : i\in I\}$ *such that, for each* $i\in I$ we have the following:

- $\mathrm{Aut}(H_i)\cong G$.
- $|H_i| = \lambda$;
- $|I| = 2^{\lambda}$;

Theorem 2. There exists an infinite group, which is not isomorphic with none of the automorphisms group of a partially mono-unary algebra.

References

[1] D. König, Theotie der endlichen und unendlichen Graphen. Kombinatorische Topologie der Streckenkomplexse, Mathematik in Monographien 16. Akademische Verlagsgesellschaft, Leipzig, 1936.

[2] A. Kipiani, On one uniform subset in $\omega_a\times\omega_a$Bulletin of Academy of Sciences of the Georgia,no. 2 (1989) (in Russian).

[3] A. Kipiani, On automorphism groups of ω-Trees, Georgian Mathematical Journal , T. 15, No. 1, 2008.